of g must coincide with that placed before ω_n and therefore will be different for each of the two terms of the solution, which is in contradiction to the original assumption. Thus it is seen that the fact of whether oscillations are decaying or diverging is not governed, in this equation, by a single sign for g. For $g > 0$, for example, oscillations always will diverge unless x_{02} is taken equal to zero, which then leaves only one arbitrary constant to satisfy two initial conditions; further, the condition $x_0 = 0$ cannot be satisfied without reduction to a trivial solution. Among other things, the situation described points out an error in Ref. 4 which was indicated to the authors by Crandall.[3]

The fact that the physical system is clearly a decaying one for positive damping makes it appear desirable to attempt to remove the anomaly just mentioned. Crandall[1,3] suggests a method of accomplishing this, the two alternatives of which, however, must be distinguished from each other. In Ref. 1, he suggests replacing $1 + ig$ by $1 + ig \operatorname{sgn}\omega$. This applies to the forced case only. The alternative method (free case) is to replace $1 + ig$ by $1 + ig \operatorname{sgn}\omega_r$. The second alternative does, in fact, assure a decaying solution for the homogeneous part of Eq. (1). However, it does not retain damping in phase with velocity. The $1 + ig$ device is, strictly speaking, correct only for the originally intended use, namely, simple harmonic motion of the form $e^{i\omega t}$, $\omega > 0$; that is, it represents damping proportional to displacement and in phase with velocity only in this case. In any other case, such as an oscillation $e^{i\omega_d t}$ with ω_d complex, for example, the velocity, $i\omega_d x$ itself clearly has components both in and 90° out of phase with $ig\omega_n^2 x$.

The use of the first alternative, $1 + ig \operatorname{sgn}\omega$, yields absurd results[1] for the response in the case where the right-hand side of Eq. (1) is replaced by a pulse and the system is analyzed employing the Fourier transform over $-\infty \leq \omega \leq \infty$. The particular absurdity that occurs when the Fourier transform is employed is a result of the artificial discontinuity introduced in g at $\omega = 0$.

In brief, therefore, the factor $1 + ig$ $(g > 0)$ works to give the desired damping effect only in the sinusoidal case intended and not in others. A modification of it is unnecessary in the original (sinusoidal) case; one suggested modification fails to provide damping proportional to displacement and in phase with velocity in the free case and exhibits other anomalies under Fourier transform treatment in nonsinusoidal forced cases. If damping proportional to displacement and in phase with velocity is desired, which will eliminate the previous anomalies in the free case (perhaps as a tour de force), the authors suggest the following:

Suppose that one is given a single degree of freedom system with undamped natural frequency $\omega_n = (k/m)^{1/2}$ and with a damping force having the properties that it is 1) proportional to the displacement, and 2) in phase with the velocity. Then the free oscillations of such a system can be described exactly by the following differential equation:

$$m\ddot{x} + (gk/\omega_d)\dot{x} + (1 + g^2)^{1/2}kx = 0 \tag{2}$$

where the damped frequency ω_d is real and is given by

$$\omega_d = \omega_n\{[1 + (1 + g^2)^{1/2}]/2\}^{1/2} \tag{3}$$

A similar formulation is given by Pinsker[5] in commenting on a paper by Soroka.[6] It can be determined readily that Eq. (2) satisfies the desired requirements precisely and does not have any of the anomalies associated with Eq. (1). It therefore provides an exact viscous model corresponding to the desired structural damping model.

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Effects of Mass Addition on the Stability of Slender Cones at Hypersonic Speeds

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THE effect of mass addition on the flow over bodies moving at hypersonic speeds has been studied by several investigators (e.g., Cresci and Libby[1]). In most of this work, primary attention logically has been directed toward the effects of foreign-gas injection on heat transfer and pressure distributions, and, principally for this reason, most of the work has been done at zero angle of attack. The foreign gas can be provided either by some active injection system or by the action of an ablation heat shield. With increasing rates of injection, the basic flow about the body can be affected significantly. One such effect was observed in the paper by Cresci and Libby,[1] where it was shown that the shock-wave standoff distance can be increased by gas injection at the nose of a body.

Another effect of mass addition at the nose has been investigated in the 14-in. helium tunnel at the NASA Ames Research Center. In this study, a cone having a semivertex angle of 10° was tested at a Mach number of 21 and at a Reynolds number (based on length) of 4.5×10^6. The cone was 2 in. in diameter at the base and had a hemispherical tip of 0.076-in. radius. In this hemispherical tip were one hole 0.040 in. in diameter and eight holes 0.025 in. in diameter. From these holes helium was injected at various rates, and the effects of this gas injection on forces and moments were determined at angles of attack up to about 14°.

Some of the results of this investigation are shown in Fig. 1. For these results, the mass rate parameter $\dot{m}$ is the ratio of the mass rate of injection to the product of the freestream velocity, freestream density, and body base area. These results show that the mass addition decreases the stability at low angles of attack and increases it at intermediate angles. In fact, a crossover occurs, and at higher angles the pitching moments are increased in magnitude compared to those for the body without injection. With increasing mass addition, the changes in stability and moments become more pronounced, and the crossover angle of attack increases. The normal force, however, decreases with increasing injection rate at all angles of attack, although the reductions are somewhat less at the higher angles. A possible explanation for this behavior can be obtained with the aid of the results of another series of tests. The same cone with a series of oversized spheres at the tip was tested at a Mach number of 18 and a Reynolds number of 3.7×10^6. The direct contribution of the sphere drag on the normal forces and pitching moments was assumed to correspond to a drag coefficient of

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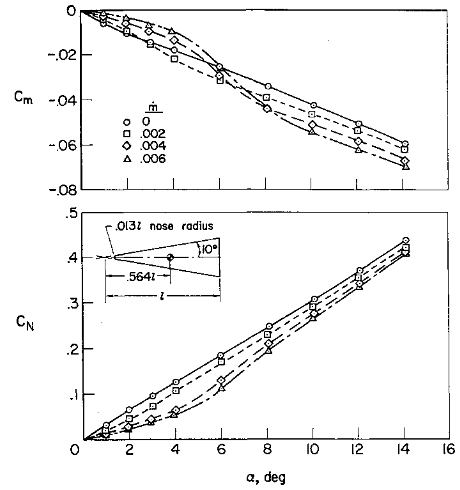

Fig. 1 Effect of mass addition on the normal-force and pitching-moment coefficients for a cone at a Mach number of 21.1 in helium

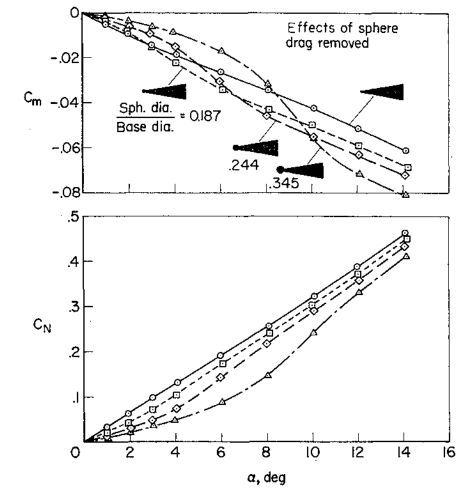

Fig. 2 Effect of oversized spherical bluntness on pitching-moments and normal-force coefficients for a cone at M = 18.1 in helium

1.0; this contribution was subtracted to obtain the results shown in Fig. 2. Note that increasing sphere size has the same effect on pitching moments and normal forces as increasing mass addition rate.

From these latter results, it is suggested that "separation-like" phenomena may be occurring as follows: the injection creates a relatively thick layer of low-energy gas which nearly encases the cone. In many respects this layer behaves as a separated region. When the cone is inclined, the layer cannot support readily the circumferential variation of pressure which produces normal loading. For this reason, both normal force and pitching moment would be reduced, at least at small angles of attack where the cone remains nearly encased in the separated layer. At somewhat higher angles, the separated or low-energy gas will collect on the lee side of the cone. Accordingly, the extent of separation on the windward side will be reduced, and in unseparated regions increased normal loading will be possible. Obviously this trend will occur at lower angles when the separated layer is thinner relative to the body radius. For the test cones, considerations of geometry and continuity indicate that the separated layer is indeed thinner relative to the radius toward the near of the body. From these considerations, it appears that the loading should return to the portions of the body aft of the moment center at lower angles of attack than it returns to the portions forward of the moment center. For this reason, it is suggested that, at some intermediate angles of attack, the moments for a body with separation may exceed in magnitude those for a body without. At these angles, however, the normal forces still will be reduced compared to those for a body without separation. These trends are, of course, those observed in the experimental results.

Use of Transient "Thin-Wall" Technique in Measuring Heat Transfer Rates in Hypersonic Separated Flows

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IN the course of an experimental investigation of hypersonic separated flows being carried out at the Gas Dynamics Laboratory in Princeton, several methods of measuring local heat transfer rates were investigated. One of these was the transient "thin-wall" technique, in which a thin-skinned model is subjected to forced convective heat transfer, and the local temperature-time history is used to obtain a measure of the heat transfer rate. This method has been used successfully for many years in the study of attached flows and, more recently, has been applied to separated flows. The model being studied is usually isothermal in the no-flow condition. The tunnel then is started, and the model temperature is recorded as it proceeds toward the recovery temperature distribution. The quantity required is the temperature-time gradient at the instant of starting the tunnel, i.e., when the model was isothermal. In most cases, the heat transfer rate to the model at any point can be expressed usefully in terms of the modified Newtonian Law, and this formulation suggests that the temperature-time variation should be exponential in character for at least the early part of a test run. As a result, an exponential temperature law, or something close to it, usually is anticipated in these experiments. In fact, one common method of reducing the data is to plot the temperature gradient against time from the tunnel start on a logarithmic ordinate scale and extrapolate back to the initial instant using a straight-line law. The reason for extrapolating back to the initial instant is, of course, to avoid the necessity of estimating the conduction effects along the model.

Initial experiments made at this laboratory with laminar cavity-type separated flows at hypersonic Mach numbers have shown that the concepts just outlined must be discarded in certain cases. The temperature-time traces from the section of the model immersed in separated flow in these experiments were very far from exponential in nature, even in the earliest stages of a run. The temperature-time gradient started with a very small value, increased to a maximum, and then decreased again after passing through a point of inflexion. In the reattachment zone and downstream, the traces showed no points of inflexion but still were not exponential in type (see Fig. 1).

Several possible causes of these peculiarities have been considered, and it has been concluded that they were the result of extreme conduction rates along the model. Apparently, the heat transfer rates in the region of separated flow were very low, and the temperature-time traces started

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